In complete sentences, using proper English and mathematical notation, state the Fundamental Theorem of Calculus (both parts) and the Net Change Theorem.

SCORE: _____ / 5 PTS

IF f IS CONTINUOUS ON $[a,b]$
AND $A(x) = \int_a^x f(t)\,dt$
THEN $A'(x) = f(x)$ FOR $x \in (a,b)$

IF f IS CONTINUOUS ON $[a,b]$
AND $F' = f$ ON $[a,b]$
THEN $\int_a^b f(x)\,dx = F(b) - F(a)$

GRADED BY ME

IF F' IS CONTINUOUS ON $[a,b]$
THEN $\int_a^b F'(x)\,dx = F(b) - F(a)$

Acme Transport Service charges by weight to move cargo in large metal containers. **SCORE: _____ / 2 PTS**

If $f(x)$ is the cost (in hundreds of dollars per ton) that Acme charges to move additional weight when a container already weighs x tons,

what is the meaning of the equation $\int_3^5 f(x)\,dx = 10$?

NOTES: **Your answer must use all three numbers from the equation, along with correct units.**
Your answer should NOT use "x", "f", "integral", "antiderivative", "rate", "change" or "derivative".
Your answer should sound like normal spoken English.

IT COSTS \$1000 MORE TO MOVE A 3 TON CONTAINER IF ITS WEIGHT INCREASES TO 5 TONS.

GRADED BY ME

Evaluate the following integrals, or explain why they can't be evaluated. **SCORE: _____ / 13 PTS**

[a] $\int \frac{3\sin 2\theta}{\sqrt{1-\cos^4\theta}}\,d\theta$

$\boxed{u=\cos^2\theta}$ 1

$du = 2\cos\theta(-\sin\theta)\,d\theta$

$= -\sin 2\theta\,d\theta$ 1/2

1/2

$-3\int \frac{1}{\sqrt{1-u^2}}\,du$ 1

1

$= -3\arcsin u + C$

$= -3\arcsin(\cos^2\theta) + C$

1 1/2

[b] $\int_{-\pi}^{\pi} \frac{t}{\sqrt[3]{1-4t^2}}\,dt$

$1-4t^2=0$ @ $t=\pm\frac{1}{2}$

INTEGRAND DISCONTINUOUS @ $t=\pm\frac{1}{2}$ 1

FTC 2 DOES NOT APPLY 1/2

[c] $\int \frac{(2-3\sqrt{y})^2}{5y^2}\,dy$

$= \int \frac{4-12y^{\frac{1}{2}}+9y}{5y^2}\,dy$

1 1/2

$= \int\left(\frac{4}{5}y^{-2} - \frac{12}{5}y^{-\frac{3}{2}} + \frac{9}{5}y^{-1}\right)dy$

$= \frac{4}{5}(-y^{-1}) - \frac{12}{5}(-2y^{-\frac{1}{2}}) + \frac{9}{5}\ln|y| + C$

$= -\frac{4}{5}y^{-1} + \frac{24}{5}y^{-\frac{1}{2}} + \frac{9}{5}\ln|y| + C$

1/2 1 1/2 1/2

MUST HAVE ABSOLUTE VALUE

[d] $\int_{-3}^{3} \frac{s^3}{s^4+2s^2+1}\,ds = 0$ 1/2

CONTINUOUS 1/2

$\frac{(-s)^3}{(-s)^4+2(-s)^2+1} = \frac{-s^3}{s^4+2s^2+1}$ ODD

1

If $L(x) = \int_{\tan^{-1} x}^{1-x^2} \sin \sqrt[3]{t}\, dt$, find $L'(x)$. **SCORE: _____ / 4 PTS**

$$L(x) = \int_{\tan^{-1}x}^{0} \sin \sqrt[3]{t}\, dt + \int_{0}^{1-x^2} \sin \sqrt[3]{t}\, dt$$

$$= -\int_{0}^{\tan^{-1}x} \sin \sqrt[3]{t}\, dt + \int_{0}^{1-x^2} \sin \sqrt[3]{t}\, dt$$

$$L'(x) = -\frac{d}{d(\tan^{-1}x)} \int_{0}^{\tan^{-1}x} \sin \sqrt[3]{t}\, dt \cdot \frac{d(\tan^{-1}x)}{dx} + \frac{d}{d(1-x^2)} \int_{0}^{1-x^2} \sin \sqrt[3]{t}\, dt \cdot \frac{d(1-x^2)}{dx}$$

$$= -\sin \sqrt[3]{\tan^{-1}x} \cdot \frac{1}{1+x^2} + \sin \sqrt[3]{1-x^2} \cdot (-2x)$$

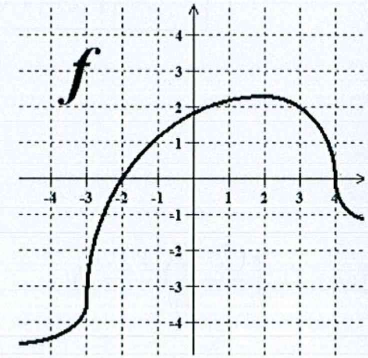
$\frac{1}{2}$ 1 $\frac{1}{2}$ 1

$$= \boxed{-}\frac{\boxed{\sin \sqrt[3]{\tan^{-1}x}}}{\boxed{1+x^2}} \boxed{-2x}\boxed{\sin \sqrt[3]{1-x^2}}$$

1

NOTE: $\sqrt[3]{\tan^{-1}x} \neq \tan^{-\frac{1}{3}}x$

Let $Q(x) = \int_{-5}^{x} f(t)\,dt$, where f is the function whose graph is shown on the right. **SCORE: _____ / 6 PTS**



[a] Write "I UNDERSTAND" to indicate that you understand that the graph shows f, but that the questions below ask about Q.

[b] Find $Q'(3)$. **Explain your answer very briefly.**

$Q' = f$ SO $Q'(3) = f(3) = 2$

1 $\frac{1}{2}$

[c] Find the x – coordinates of all critical points (ie. critical numbers) of Q. **Explain your answer very briefly.**

$Q' = 0$ WHEN $f = 0$ @ $x = -2, 4$

1 $\frac{1}{2}$ $\frac{1}{2}$

[d] Find all intervals over which Q is both decreasing and concave up at the same time. **Explain your answer very briefly.**

$Q' < 0$ AND INCREASING WHEN $f < 0$ AND INCREASING

$\frac{1}{2}$ 1

ON $(-5, -2)$ 1